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POSSIBILITY OF IDENTIFYING THE VISCOPLASTIC PROPERTIES OF LIQUIDS IN EXPERIMENTS WITH AN OSCILLATING-CUP VISCOMETER

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Results of a numerical solution of the problem of oscillations of an oscillating-cup viscometer filled with a viscoplastic liquid are presented. It is shown that near the rotation axis, a stagnant zone arises, whose boundary changes position during the oscillations. The effect of the plastic properties of the liquid on the frequency and damping coefficient of oscillations of the viscometer is determined. A method of identifying the viscoplastic properties using observed oscillation parameters is proposed.

Key words: *non-Newtonian liquids, viscoplastic liquid, oscillating-cup viscometer.*

Introduction. The torsional oscillation method is widely used in practice, in particular, in studies of internal friction in condensed media. In the physics of liquids, in particular, aggressive ones (metallic melts and salt melts), this method is the main method used in viscosity measurements. One of its main advantages is that it measures the oscillation period and damping coefficient with accuracy difficult or impossible to reach with other techniques. This allows this method to be implemented as an absolute one if the problem of the relationship between measured parameters and liquid parameters can be solved with sufficient degree of accuracy. At present, this problem has been solved analytically only for Newtonian [1] or viscoelastic [2] liquids. Most experimental data obtained with an oscillating-cup viscometer have been interpreted assuming Newtonian behavior of the liquid studied. The possibility of using this method to study the rheological properties of different liquids have been studied inadequately. In particular, it is not clear how much the measured oscillation parameters change if the properties of the liquid differ from Newtonian ones. Of special interest is the situation where the difference is insignificant and the data are interpreted using the Newtonian approximation. For example, Apakashev and Pavlov [3] found the elastic properties of water — a liquid regarded as a classical Newtonian medium. In addition, inaccurate rheological equations may be responsible for the large spread of experimental data on the viscosity of another class of Newtonian liquids — metallic melts. In this connection, estimating the oscillation parameters of a viscometer filled with a medium with weakly expressed non-Newtonian properties seems an important problem.

In the present paper, we consider a viscometer model filled with a viscoplastic liquid — a medium whose flow becomes possible only after the shear stress exceeds a certain threshold — the yield limit. In view of the well-known mathematical problems that arise in describing even simple flows of viscoplastic liquids [4], at the present stage of the study, we confine ourselves to a consideration of the simplest model — a linear viscoplastic medium.

Mathematical Model. We consider a cylindrical vessel suspended by an elastic thread. The vessel has height H, inner radius $R_{\rm c}$, and moment of inertia $I_{\rm c}$, is filled with a viscoplastic liquid, and performs damping torsional oscillations around its axis. It is required to determine the dependence of the frequency f and damping coefficient p of oscillations of the cylinder on the viscoplastic properties of the liquid. In the approximation of an infinitely long cylinder and under the assumption of axial symmetry of the flow, only the azimuthal component of the liquid velocity V_φ is different from zero. In this case, the equation of motion of the viscoplastic medium in the viscous flow region is written in cylindrical coordinates as

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$$\frac{\partial V_\varphi}{\partial t} = -\frac{1}{\rho}\Big(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\tau_{r\varphi})\Big), \quad (1)$$

where t is time, ρ is the density of the liquid, and $\tau_{r\varphi}$ is the stress tensor component. For a linear viscoplastic material, the following relations are valid [5]:

$$\tau_{ij} = -(\eta + \tau_0/\sqrt{|(\dot\varepsilon,\dot\varepsilon)/2|})\dot\varepsilon_{ij}, \qquad \sqrt{(\tau,\tau)/2} > \tau_0; \quad (2)$$

$$\dot\varepsilon_{ij} = 0, \qquad \sqrt{(\tau,\tau)/2} \leqslant \tau_0. \quad (3)$$

Here η is the dynamic viscosity, τ_0 is the yield limit, τ_{ij} and $\dot\varepsilon_{ij}$ are the stress tensor and strain rate components, among which only the components $\tau_{r\varphi}$ and $\dot\varepsilon_{r\varphi} = \partial V_\varphi/\partial r - V_\varphi/r$ are different from zero in the adopted approximation. Equation (1) is valid only in the region where the viscous flow conditions (2) are satisfied, and the remaining part of the liquid moves as a rigid body (forms a stagnant zone). Thus, in the adopted approximation, the stagnant zone has the shape of a cylinder (if it is at the center of the viscometer) or a cylindrical layer of a certain thickness. Then, the equations of motion for the viscometer cylinder and the stagnant zones become

$$I_{\rm c}\frac{d\omega_{\rm c}}{dt} = -\varkappa\varphi + M_{\rm c}; \quad (4)$$

$$I_i\frac{d\omega_i}{dt} = M_i, \quad (5)$$

where $\varkappa$ is the torsional stiffness of the thread, φ is the rotation angle of the cylinder, I_i and ω_i are the moment of inertia and the angular velocity of the stagnant zone, $\omega_{\rm c}$ is the angular velocity of the cylinder, and $M_{\rm c}$ and M_i are the moments of the friction forces exerted on the cylinder and the stagnant zone by the liquid; the subscript i corresponds to the zone number starting from the axis of the viscometer cylinder. In (5), it is assumed that several stagnant zones can exist simultaneously.

As the boundary conditions we use slip conditions on the solid boundaries:

$$V_\varphi(R_{\rm c},t) = \omega_{\rm c}R_{\rm c}, \quad V_\varphi(R_{{\rm ext},i},t) = \omega_i R_{{\rm ext},i}, \quad V_\varphi(R_{{\rm int},i},t) = \omega_i R_{{\rm int},i}. \quad (6)$$

Here $R_{{\rm ext},i}$ and $R_{{\rm int},i}$ are the external and internal radii of the ith stagnant zone, respectively. Before the beginning of motion, the cylinder with the liquid is at rest and is rotated through a certain angle φ_0 about the equilibrium position. At the time $t = 0$, the cylinder is released. In this case, the initial conditions are written as

$$V_\varphi(r,0) = 0, \qquad \varphi(0) = \varphi_0, \qquad \omega_{\rm c}(0) = 0. \quad (7)$$

A numerical solution is found by a finite-difference method. Equations (1)–(7) are made dimensionless by normalizing the distanced by the radius $R_{\rm c}$, the rates by $\nu/R_{\rm c}$ (ν is the kinematic viscosity of the liquid), the pressure by $\rho\nu^2/R_{\rm c}^2$ (ρ is the density of the liquid), and time by $R_{\rm c}^2/\nu$. A uniform grid with the maximum partition of up to 2000 steps in the radial direction is used. The spatial derivatives are discretized using a central difference scheme with accuracy up to $(\Delta x)^2$, and the time derivatives are discretized using a unilateral scheme with accuracy up to Δt. The solution of the linearized systems of equations in each time step is performed using a marching method [6]. The solution of system (1)–(7) at the time t^{n+1} is obtained using the previous (determined in the previous time step) position of the boundary separating the region in which the liquid moves as a rigid body from the viscous flow region. In view of a finite accuracy of the numerical scheme, the difference analog of the constitutive relations (2) is written as follows:

— In the viscous flow region,

$$\sqrt{(\dot\varepsilon,\dot\varepsilon)/2} \geqslant \delta;$$

— In the stagnant zones,

$$\sqrt{(\dot\varepsilon,\dot\varepsilon)/2} < \delta$$

(δ is a small parameter). Preliminary calculations showed that $\delta = \tau_0 R_{\rm c}^2/(N\nu^2\rho)$ (N is the number of spatial partitions).

The calculations were performed for the following values of the parameters of the viscometer and viscoplastic medium: cylinder radius $R_{\rm c} = 0.01$ m, moment of inertia $I_{\rm c} = 3I_{\rm liq}$ ($I_{\rm liq}$ is the moment of inertia of the "frozen"

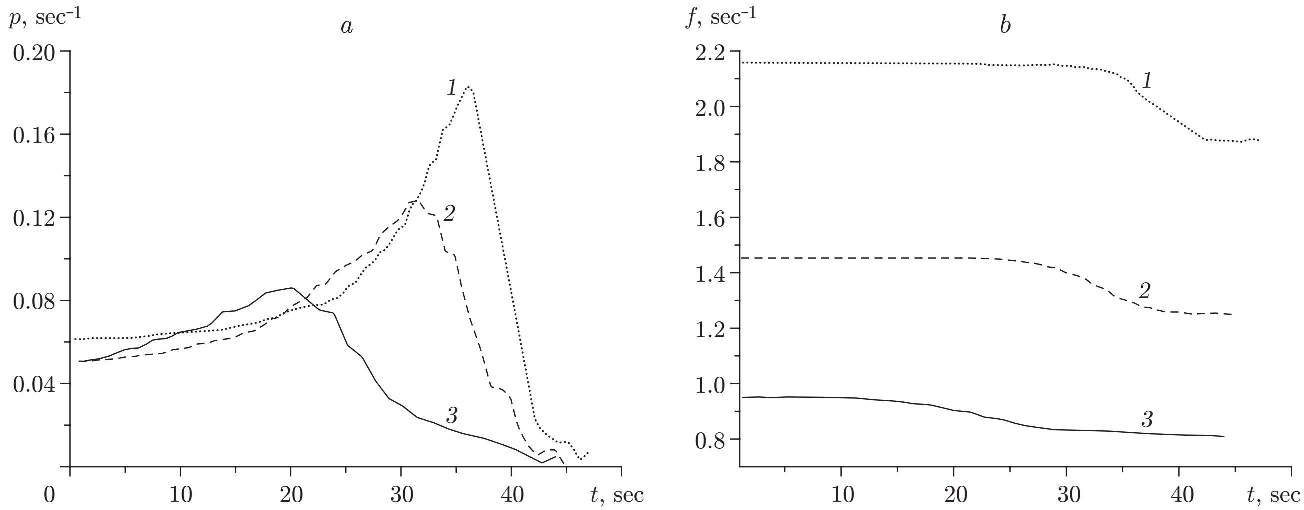

Fig. 1. Damping coefficient (a) and frequency (b) of oscillations of the viscometer versus time for various values of the torsional oscillation period: $T = 2.90$ (1), 4.18 (2), and 6.61 sec (3); $\tau_0 = 10^{-4}$ Pa, $R_{\rm c} = 0.01$ m, and $I_{\rm c}/I_{\rm liq} = 3$.

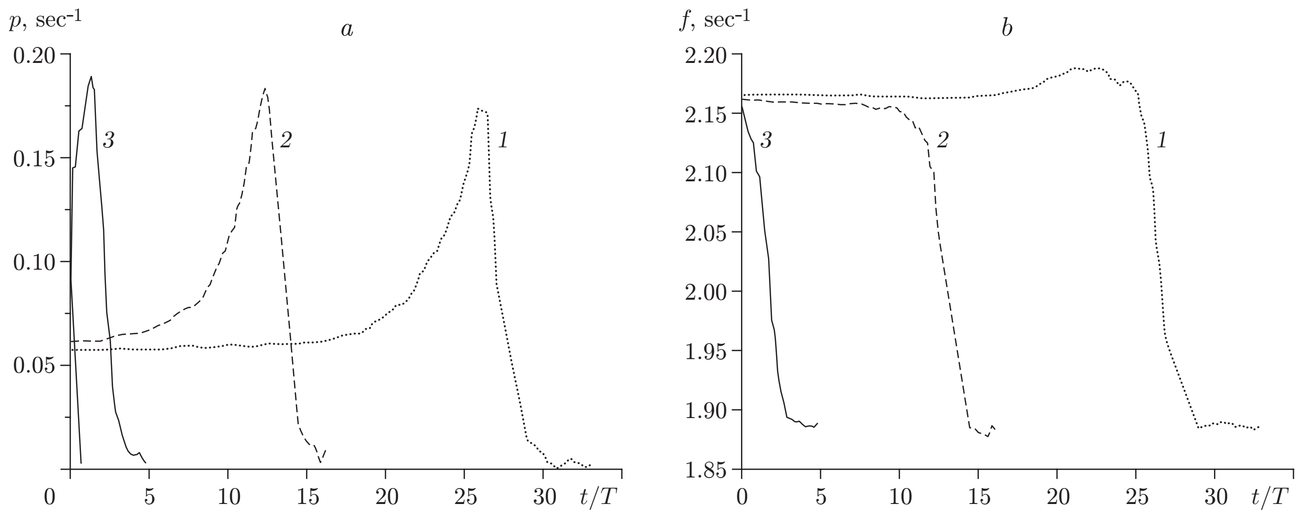

Fig. 2. Damping coefficient (a) and frequency (b) of oscillations of the viscometer versus the numbers of oscillation for various values of the yield limit of the liquid: $\tau_0 = 10^{-5}$ (1), 10^{-4} (2), and 10^{-3} Pa (3); $T = 2.9$ sec, $R_{\rm c} = 0.01$ m, and $I_{\rm c}/I_{\rm liq} = 3$.

liquid); the torsional stiffness of the thread was such that the initial oscillation period was 2.9, 4.18, or 6.61 sec. The viscosity and density of the liquid corresponded to water. The yield limit was small: $\tau_0 \leqslant 10^{-3}$ Pa.

Results and Discussion. As shown by the calculations, for small-amplitude oscillations, the strain rate is different from zero only in the region adjacent to the cylinder surface. A stagnant zone that does not disappear with time can form along the axis. During oscillation damping, the radius of the axial stagnant zone increases on the average and, at a certain time, it has an influence on the cylinder oscillations and the frequency and damping coefficient of oscillations of the viscometer.

Figure 1 gives time dependences of the damping coefficient and frequency of oscillations of the viscometer for various values of the oscillation period T; the latter is easy to vary in experiments by changing the length of the suspension thread. To obtain curves 1–3, we approximated the law of motion of the viscometer $\varphi(t)$ by the function

$$\varphi(t) = A\,{\rm e}^{-pt} \sin{(ft + \psi)},$$

where f is the oscillation frequency and p is the damping coefficient. The approximation was performed by the least squares method with minimization using the Rosenbrock algorithm [7]. The oscillation characteristics were determined locally from data for a small segment of the oscillation record. The time dependence of the oscillation characteristics of the viscometer (Fig. 1) has three main segments. The first segment is characterized by constant values of p and f. On the second segment, the damping coefficient increases and the oscillation frequency decreases, which is due to the overlapping of the liquid boundary layers adherent to the stagnant zone and the viscometer cylinder. In this case, the damping coefficient and frequency change by a value sufficient to be recorded in experiments with an oscillating-cup viscometer, in which it is possible to reach a relative measurement error of about 10^{-4}. The third segment of the oscillation propagation is characterized by a sharp reduction in the damping coefficient to almost zero. This is due to the fact that the radius of the axial stagnant zone increases to such an extent that it can adhere to the cylinder. In this case, the entire system moves as a rigid body. The damping is determined by the schematic viscosity and, hence, it is small, the oscillation frequency remains constant since the moment of inertia of the system does not change. In the case of adhesion of the stagnant zones to the cylinder, the moment of inertia becomes the largest and is equal to the sum of the moments of inertia of the cylinder and the liquid "frozen" in it.

In Fig. 1, it is evident that the division into the segments is conditional. The change in the oscillation damping coefficient and frequency are the more significant the smaller the oscillation period. When choosing a very small period (a high torsional stiffness of the suspension thread) in the experiment, it is necessary to take into account the following: as the period decreases, the stationary segment of the curves of $p(t)$ and $f(t)$ increases and, hence, the oscillation amplitude is smaller at the time when the damping coefficient begins to change. This can cause difficulties in separating a very small signal (decaying sinusoid) from noise.

The calculations show that, other things being equal, the change of the torsional oscillation regimes (stationary regime–increasing damping–decreasing damping) is the faster the higher the yield limit of the viscoplastic medium. Figure 2 gives curves of the damping coefficient and oscillation frequency versus oscillation number for liquids with various yield limits. For $\tau_0 = 10^{-5}$ Pa, the maximum of the curve of $p(t)$ and the corresponding jump of the frequency $f(t)$ are observed at the 27th oscillation, whereas for $\tau_0 = 10^{-4}$ Pa, it is observed at the 13th oscillation. For a liquid with an yield limit $\tau_0 = 10^{-3}$ Pa, the curves of $p(t)$ and $f(t)$ do not have a stationary segment, i.e., the dimensions of the stagnant zone are such that it has a significant effect on the oscillation parameters from the beginning of the process.

Thus, the examined features of the time dependences of the oscillation parameters can be used to identify a liquid as a viscoplastic one and to perform a comparative analysis of the yield limit of various viscoplastic media.

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